# Measuring Constructive Alignment:
# An Alignment Metric to Guide Good Practice

Dr Jonathan A. Tepper
School of Computing & Informatics
Nottingham Trent University
Clifton Campus, Clifton Lane
Nottingham, NG11 8NS
Jonathan.Tepper@ntu.ac.uk

## ABSTRACT

*We present a computational model that represents and computes the level to which an educational design is constructively aligned. The model is able to provide 'alignment metrics' for both holistic and individual aspects of a programme or module design.*

*A systemic and structural perspective of teaching and learning underpins the design of the computational model whereby Bloom's taxonomy is used as a basis for categorising the core components of a teaching system and some basic principles of generative linguistics are borrowed for representing alignment structures and relationships. The degree of alignment is computed using Set theory and linear algebra.*

*The model presented forms the main processing framework of a software tool currently being developed to facilitate teachers to systematically and consistently produce constructively aligned programmes of teaching and learning. It is envisaged that the model will have broad appeal as it allows the quality of educational designs to be measured and works on the principle of 'practice techniques' and 'learning elicited' as opposed to content.*

### Keywords

*Constructive alignment, alignment metric, alignment tree, learning outcome, Bloom's taxonomy.*

## 1. INTRODUCTION

Designing learning is considered one of the most fundamental activities of a teaching practitioner and the aim of such a design process is to assist in the development of conscious and purposeful teaching and learning [10]. Learning is '*…a cognitive activity that involves the use of intellect for the development and structuring of understanding about oneself and the world in which one lives*' (Wilson [24]). Good teaching practice is commonly perceived as that which creates and uses learning environments and activities that fosters *deep student learning* [17]. Deep learning occurs when the student is motivated to understand and engage with the concepts taught to satisfy intrinsic curiosity and to attain higher conceptual levels of understanding [17,4].

Since 1992, there has been a significant growth in universities, programmes and students within the United Kingdom (UK). Approximately 44% of young people within the UK are currently experiencing higher education (HE) [11]. The perceived continual introduction of new change initiatives within UK higher education institutions (HEIs) to meet government targets creates a dynamic and challenging environment for practitioners to 'ply their trade'. That is for teachers to create the appropriate teaching and learning environment(s) to consciously meet the academic needs of *all* of their students. For example, the government's agenda for widening participation and fair access [13,18] encourages universities to widen access to their programmes by improving provision for the admission and retention of students from non-traditional backgrounds. These may include those from lower socioeconomic groups, students with non-traditional HE qualifications, disabled students and certain minority ethnic groups. So in addition to accommodating larger class sizes caused by educational rationalism, it is important for the teacher to also meet the challenges associated with increased variability in student academic ability.

This is at a time when programme specifications are coming to the fore of public scrutiny and as the government allows universities to charge up to £3,000 in tuition fees from September 2006 [9]. The recent inclusion of programme specifications on the publicly available Teaching Quality Information (TQI) website enables students to quite rightly make comparisons of programmes and related quality information across HEIs in order to make an informed choice. In the resulting HE market that is likely to ensue from increased fees, it would be reasonable to assume that fee-paying students will naturally become more 'outcome aware'. That is, to not only assess the new academic knowledge and understanding potentially offered by a course but to also assess *how* the supplying university can ensure, through its teaching and learning practices and resources, that such intended learning outcomes can be potentially achieved by all students admitted. This is potentially significant, as universities will clearly become more accountable for the correlation between the promotion and delivery of their degree programmes. Failure to ensure a teaching and learning environment that is perceived as fair and adequate by *both* student and HEI could lead to costly litigation proceedings as evidenced in 2002 [1].

The 'big bang' for HE sector-wide adoption of the outcome-based learning approach to educational design was apparent after The Dearing Report [18] accepted the findings and recommendations made by Higher Education Quality Council (HEQC) [14]. Dearing subsequently recommended that learning intentions were to be expressed as learning outcomes in programme specifications. Wide-scale adoption is now clearly evident with intended learning outcomes now considered to be the basis for standards (see [12]) and a common framework for credit accumulation and transfer within HE. *It is therefore essential within the current HE climate that teaching practitioners are able to construct and articulate their educational designs (and thus that of student learning) within an outcomes-based context*.

This paper presents a computational model that enables teaching practitioners to quantitatively measure how well-aligned their educational design is with respect to the intended learning outcomes. More specifically, the model represents and computes the level of *constructive alignment* [3]. Constructive alignment is an outcome-based methodology developed by John Biggs for designing, promoting and assessing deep student learning. It works on the fundamental assumptions that:

- Students constructs his or her own learning through relevant learning activities;
- Teachers create a learning environment that supports learning activities appropriate to achieving the desired outcomes.

Constructive alignment marries the main tenets of constructivism (see [25]) and instructional design [8] to ensure that not only are the assessment tasks aligned with the learning outcomes, but so too are the teaching and learning activities in which a student engages. It is a widely accepted outcomes-based approach that can facilitate teaching practitioners to meet some of the challenges associated with programme and module development within HE today.

## 2. A COMPUTATIONAL MODEL FOR MEASURING CONSTRUCTIVE ALIGNMENT

A fundamental aim of this research is to develop a so-called 'alignment metric' to assist the teaching practitioner during the curriculum design, construction and improvement process. The following research question initiated the chosen line of inquiry:

> *How can we quantitatively measure the level to which a programme or module design is constructively aligned?*

Naturally, this provoked further questions, relating to the efficacy of a metric tool that could measure the level of constructive alignment. More specifically, it was asked whether such a tool could facilitate teaching practitioners to:

i. adapt their practice to improve the alignment of their module designs?;
ii. design and develop constructively aligned curricula that is fair to all students and enforces inclusivity?

This section first enumerates the theoretical motivations, which enables these questions to be addressed. A computational model of constructive alignment is then presented using set theory to represent component relations and linear algebra to represent and compute alignment.

### *Theoretical Motivations*

Constructive alignment, through its integration of instructional design and constructivist principles, offers a theoretical and practically proven alignment system (see [5]) that can form the basis of a computational system engineered to assist the teacher during curriculum design. It is hypothesised that such a computational system is realisable on the premise that the desiderata for representing and computing alignment are:

- adopt a systemic and structural view of educational design;
- categorise system components according to the level of cognitive ability they elicit from the student;
- apply set theory and linear algebra to express, represent and compute alignment.

The motivations for each of the above important factors will be briefly considered.

## 2.1 A Systemic and Structural Perspective

Teaching can be thought of as a system and an important characteristic of all systems is the interactions between system components to achieve a common goal or stable state i.e. equilibrium. We adopt Biggs's [2] systemic view of teaching and learning within tertiary education. Although, Biggs [2] identifies several nested micro-systems existing within the tertiary education system, we focus on Biggs's *classroom system* which has component parts comprising of students, teachers, and teaching context. Equilibrium occurs within this system when there is a convergence of agreement between the teacher's perceptions of student competences and curriculum needs, setting of tasks, students' perceptions of task demands, teaching and learning processes, and learning outcomes. If a misalignment between these components occurs, e.g. between students' perception of task demands and of teaching processes, then low level outcomes or collaborative student misconceptions could result.

To extend this hypothesis to alignment systems and in particular constructive alignment, we assert that Biggs's classroom system should inherently embody constructive alignment in the sense that each component of an aligned teaching and learning design is operationalised by and communicated between components of the classroom system. It should therefore be considered to be an open system that is subject to change in order to adapt its behaviour towards a more stable state. A stable state represents a convergent perspective of the learning and teaching environment that is driven by explicit alignment activities. Such changes may have occurred from the result of defining or modifying learning objectives to elicit cognitive abilities better suited to the learning outcome(s) they are associated or a change in a TLA to better suit one or more learning objectives or assessment tasks. Both teacher and student are adaptive agents within this process. The teacher (and even the student) may be the initiator of the change to attain better alignment. The causal effect of this change is hopefully for students to adapt their perspectives and approaches to learning and thus learning in a more appropriate manner. Subsequently, when the classroom system reaches an equilibrium state so too does a system based on constructive alignment. Likewise, a misalignment between components within a constructive alignment system will also lead to disequilibrium in a classroom system.

Alignment systems can also be thought of as being *structural* and *generative*. Before elaborating on this perspective further, it is first important to briefly clarify what the inter-related components of an alignment system are. When designing undergraduate programmes in UK HEIs, the main components of the educational framework are considered to be the *learning aims*, *learning outcomes*, *learning objectives*, *teaching and learning activities (TLAs),* and *assessment tasks (ATs)*. Learning aims, as clarified by [23], are statements of learning which tend to be generalised. It essentially identifies the learning intentions i.e. what the teacher intends the student to learn[1]. Learning objectives are considered to be teacher-orientated and specify what it is the teacher wants the student to achieve (in terms of levels of understanding of given topics) and underpins the teaching and learning activities they subsequently prescribe [10]. TLAs are those teaching methods and techniques that are chosen to get the students to do what the learning outcomes nominate [3,4]. Biggs does not differentiate between learning outcomes and learning objectives and refers to learning objectives as being intended learning outcomes. We, however, adopt D'Andrea's [10] perspective of learning objectives as being the input to the TLAs and the learning outcomes as referring to the output of the TLAs. Intended learning outcomes are knowledge, skills and competencies the teacher hopes the students to have attained, whereas actual learning outcomes refer to those that the students actually have attained having completed the TLAs. This general view of learning objectives as input specifications and learning outcomes as the outputs or product of the student learning activities are congruent and thus hold amongst other academic viewpoints such as [19,23]. Summative ATs refer to those student activities usually prescribed by the teacher to make official judgments in relation to student academic performance on which awards are based.

Considering the above system components further, we assert that a computational model that is able to represent and measure constructive alignment can be realised as a top-down generative system that generates compatible or *aligned* teaching and learning tree structures (both whole and partial structures) in response to each learning outcome. A structural and generative model is motivated in part by the principles of syntactic theory and that of Chomsky's [7] generative linguistic theories (see [7,21]). As with generative linguistics, which requires a grammar consisting of production rules that when applied describe well-formed syntactic constructions, the generation process requires executable rules based on the principles of constructive alignment. The model

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[1] Due to the generalised nature of learning aims they are not considered further.

presented in this paper simply requires that the four system components identified can be categorised according to the cognitive ability they elicit and on this basis make dependency relations across component groups to form structure. Linear algebraic operations can then operate across structures to compute alignment.

To understand this structural perspective further, assume that our generative system can only generate three different types of tree structures: a) learning outcome ($L^o$) trees; b) learning objective ($L^b$) trees; and c) assessment task (AT) trees. Tree structures have two important properties we must consider, that is *dominance* and *valence*. Dominance refers to the parent/child relationships between nodes (system components) within the tree. For our purposes, a teacher may define a number of learning objectives (or $L^b$s) for each learning outcome ($L^o$) thus $L^o$ trees will have learning outcomes (parents) that dominate learning objectives (children). Likewise, since one or more TLAs are employed to stimulate the student to meet a learning objective, $L^b$ trees will subsequently have learning objectives that dominate TLAs. Similarly, an AT may address one or more learning objectives, thus AT trees will have assessment tasks that dominate learning objectives. The three different tree types are shown in Figure 1.

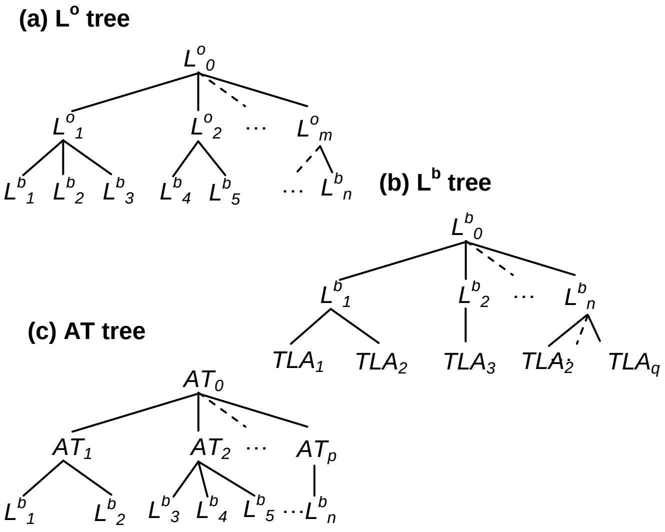

Figure 1. a) $L^o$ tree showing relationships between outcomes and objectives; b) $L^b$ tree showing relationships between objectives and TLAs and c) AT tree showing relationships between ATs and objectives.

Valence, on the other hand, refers to the number of children each parent can dominate i.e. a parent's power of dominance. For example, a teaching practitioner may define three objectives ($L^b$s) for outcome 1 ($L^o_1$) and two for $L^o_2$. Likewise, to enable a student to achieve $L^b_1$ to $L^b_3$, the teacher may ascribe $TLA_1$ and $TLA_2$ and for $L^b_4$ and $L^b_5$, $TLA_3$ may be ascribed. This variation effectively causes trees to become imbalanced and this is particularly difficult to model within fixed width vectors and matrices.

This problem can be alleviated by balancing the trees via fixing the valence for each tree type. For example, assume a teacher, within their module design, has defined *m* learning outcomes, *n* learning objectives, *p* ATs and *q* TLAs.

To balance each of the different tree types we introduce three constants, $c_1$, $c_2$ and $c_3$, whose value determines the valence and thus number of children each parent must dominate. If there are not enough children available then 'filler' elements, referred to as <*empty*> nodes in the context of trees, must be ascribed to make up the required number. The values of these constants need to be empirically established. For our purposes, assume that $c_1$ corresponds to $L^o$ trees and is fixed at 3, $c_2$ corresponds to $L^b$ trees and is fixed at 2 and finally, $c_3$ corresponds to AT trees and is fixed at 4.

When considered holistically and for an entire module or programme, such a generative system would, given the learning outcomes, generate and coordinate only those $L^o$ trees that dominate aligned $L^b$ nodes that can help, either individually or collectively, the student to meet the associated leinarng outcome.

Subsequently, the learning objectives would generate only the subset of TLAs that collectively elicit the type of student learning required by the learning objective(s). AT structures would then be generated to align and thus dominate one or more $L^b$s. Clearly there may be more than one path or structure from a given learning outcome to a given set of adequate TLAs. Each different structure can be referred to as a *derivation*. A balanced tree structure generated for a single learning outcome is shown in Figure 2.

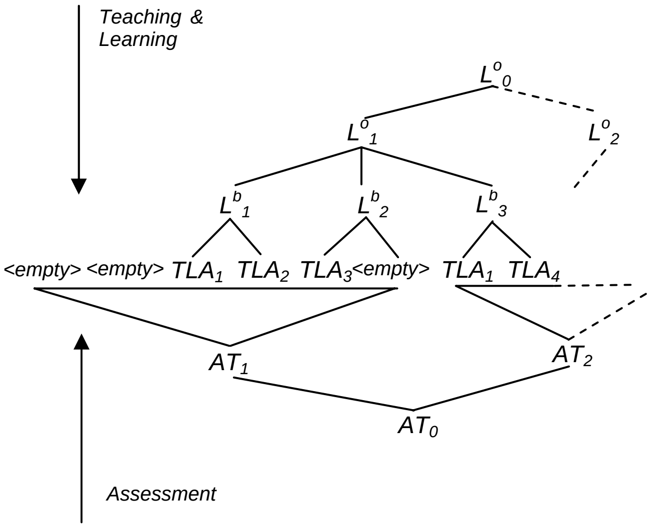

Figure 2. A balanced tree structure showing relationships between system components for a single learning outcome.

Note that in Figure 2, AT trees are pictured as dominating TLA nodes even though they actually dominate $L^b$ nodes. The reason for this is that since ATs dominate $L^b$s and $L^b$ trees dominate TLAs then AT trees indirectly dominate the TLA nodes associated with the $L^b$ nodes it directly dominates.

## 2.2 Categorising System Components using Bloom's Taxonomy

Although there is no universally accepted method of aligning elements between the four sets defined above, numerous academics have suggested possible strategies. In particular, Biggs [4] cites [20] and [22] for utilising verb-matching schemes to determine the level of cognitive ability afforded by an assessment task and provides a comprehensive list of suitable assessment tasks for the different types and levels of learning required by a learning outcome [4]. For our purposes, such verb-matching schemes make allocating each learning outcome and related set of learning objectives an appropriate level of cognitive skill elicited a relatively simple task. The level is obtained by matching the main verb in the outcome or objective with the corresponding entry in Bloom's taxonomy [6] that contains a matching or synonymous verb. Figure 3 shows each of the six levels representing levels of cognitive ability stimulated by a particular action. Level 6 refers to the *highest* cognitive ability stimulated and level 1 to the *lowest*.

| Level | Cognitive Ability Stimulated | Action Elicited |
|---|---|---|
| 6 | Evaluation | Ability to make a judgment of the worth of something. |
| 5 | Synthesis | Ability to combine separate parts into a whole. |
| 4 | Analysis | Ability to divide a problem into its constituent parts and establish the relationship between each one. |
| 3 | Application | Ability to apply rephrased knowledge to novel situations. |
| 2 | Comprehension | Ability to rephrase knowledge. |
| 1 | Knowledge | That which can be recalled. |

Figure 3. Bloom's taxonomy of learning objectives containing six levels of learning stimulated as described by Bloom [6]. Table adapted from [10].

It is slightly more difficult, however, to allocate an appropriate level of cognitive skill stimulated by a TLA because there is no such verb defined. Biggs [4] attempts to bridge this gap by defining and tabling the types of learning elicited by each type of TLA as shown in Appendix A. Biggs's motivation here is to ensure that the selection of a TLA can be governed by a set of learning outcomes rather than the TLA governing the outcome(s). Consequently, this allows the same verb-matching scheme to be used to associate each TLA with a corresponding entry in Bloom's taxonomy. Biggs also provides similar classification for ATs as shown in Appendix A, which includes classifications we have assigned based on our understanding of Bloom's taxonomy and the information provided in Biggs [4]. The classification schemes for ATs and TLAs presented by Biggs, however, is broad and ambiguous. Biggs acknowledges this by emphasising that such research is unfinished. This is therefore a significant constraint on the computational model presented.

The verb-matching schemes collectively outlined in Biggs [4] will be used as a basis to cluster the different system components according to the cognitive skill elicited.

## 2.3 Sets for Expressing Component Relations

The mechanics of linear algebra [15], through its vectors and matrices and associated mathematical operators enables us to numerically represent learning outcomes, objectives, TLAs and ATs and the relationships between them. As discussed in detail in subsequent sections, its operators allow us to perform computations across these structures to yield alignment figures for an entire programme, module or between individual components (e.g., alignment between learning objectives and TLAs). Initially, however, Set theory[2] is used to express the direct and indirect relationships that exist between system components.

The four major components of an educational design defined above can be viewed as forming four distinct sets of ordered elements. For example, assume W represents the set of all possible learning outcome declarations for a module, where each declaration, or element w, contains an active verb. The formal declaration for each of the major components is as follows:

W = {w: w is a learning outcome declaration, w contains an active verb}

X = {x: x is a learning objective declaration, x contains an active verb}

Y = {y: y is an assessment task (AT)}

Z = {z: z is a teaching/learning activity (TLA)}

Each of the above sets is considered finite for each programme or module design and thus

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[2] For a concise introduction to Set theory see [16].

contains a fixed number of elements. As shown below, the notation *n*(A) (e.g. the number of elements in set A) is used to denote the cardinality of each of the disjoint sets:

*n*(W) = m i.e. W consists of *m* learning outcomes
*n*(X) = n i.e. X consists of *n* learning objectives
*n*(Y) = p i.e. Y consists of *p* ATs
*n*(Z) = q i.e. Z consists of *q* TLAs

When each element of the four sets, W, X, Y and Z can be associated with a corresponding level in Bloom's taxonomy, a further four sets can be defined to store these levels:

$W' = \{ w' : w'$ is the level in Blooms taxonomy referenced by element w in W $1 \le x \le 6\}$
$X' = \{x' : x'$ is the level in Blooms taxonomy referenced by element x in X $1 \le x \le 6\}$
$Y' = \{y' : y'$ is the level in Blooms taxonomy referenced by element y in Y $1 \le x \le 6\}$
$Z' = \{z' : z'$ is the level in Blooms taxonomy referenced by element z in Z $1 \le x \le 6\}$

A number of corresponding relations can then be defined to associate each element of W, X, Y and Z with its corresponding element in $W'$, $X'$, $Y'$ or $Z'$. For example, suppose R is a relation from W to $W'$ then R is a set of ordered pairs where each first element comes from W and each second element comes from $W'$. That is, for each pair, w belongs to W (written $w \in W$) and $w'$ belongs to $W'$ (written $w' \in W'$), such that when $(w, w') \in R$ we say that w is R-related to $w'$, written $wRw'$. Subsequently, the following defines all relationships between the four component sets and their corresponding set representing levels from Bloom's taxonomy:

- for each pair $w \in W$ and $w' \in W'$, $(w, w') \in R$ i.e. $wRw'$
- for each pair $x \in X$ and $x' \in X'$, $(x, x') \in S$ i.e. $xSx'$
- for each pair $y \in Y$ and $y' \in Y'$, $(y, y') \in T$ i.e. $yTy'$
- for each pair $z \in Z$ and $z' \in Z'$, $(z, z') \in U$ i.e. $zUz'$

All relations represent one-to-one mappings between sets and enable us to map a teacher's original textual definitions for each component to a number representing a level in Bloom's taxonomy. Also, the *domain* of a relation is the set of all first elements of the ordered pairs (e.g., w for relation R above) and the *range* of the relation is the set of second elements (e.g., $w'$ for relation R above).

Now that the relationships between the main components of the teaching system and Bloom's taxonomy have been formally established it is now possible to group ordered pairs, across relationship types with respect to the values of $w'$, $x'$, $y'$, and $z'$. Using the basic principles of set theory this is an easy concept to realise. For example, assume that V refers to a non-empty set containing *all* elements of relations R, S, T and U, that is the 'union' (denoted by the $\cup$ operator) of relations R, S, T and U, written as $V = R \cup S \cup T \cup U$. The number of elements in V is easily determined:

$$n(V) = n(R) + n(S) + n(T) + n(U) = m + n + p + q$$
.

Partitions of V can be formed based on the type of learning elicited by each element of each relation. Since there are six levels in Bloom's taxonomy, there will be 6 non-overlapping, non-empty subsets. More precisely, a *partition* of V is a collection $\{A_i\}$ of nonempty subsets of V such that:

- Each *a* in V belongs to one of the $A_i$.
- The sets of $\{ A_i\}$ are mutually disjoint; that is, elements in $A_i$ do not occur in $A_j$ (written as $A_i \neq A_j$) thus if we attempted to form a set consisting of only those elements that occur in $A_i$ AND $A_j$ (written as $A_i \cap A_j$) we would have an empty set (written as $A_i \cap A_j = \phi$).

The subsets in a partition are called *cells*. In total, there are six disjoint cells, $A_1$, $A_2$ , $A_3$ , $A_4$ , $A_5$ , and $A_6$. Clearly, such well defined partitions assume that it is possible to unambiguously cluster elements of V using some categorisation or matching technique as discussed earlier. The contents in each cell, $A_i$ , of V would therefore contain *subsets* of relations R, S, T and U where the index *i* refers to the level addressed in the Bloom's taxonomy by the associated learning outcome, learning objective, AT or TLA. Assuming that $R_i$ is a subset of R, such a relationship is formally written as $R_i \subseteq R$. The formal definitions specifying the contents in each cell of $A_i$ are as follows:

$R_i \subseteq R$ and $(w, w') \in R_i$ if and only if (iif) $w' = i$

$S_i \subseteq S$ and $(x, x') \in S_i$ iif $x' = i$

$T_i \subseteq T$ and $(y, y') \in T_i$ iif $y' = i$

$U_i \subseteq U$ and $(z, z') \in U_i$ iif $z' = i$

We assert that this represents the type of grouping that teaching practitioners should be attempting to perform during the module (or programme) construction process in order to obtain constructively aligned modules.

## 2.4 Vectors and Matrices for Representing and Computing Alignment

The partition of V into six disjoint sets of aligned component elements represents the ideal selections from the teacher's repertoire given a set of learning outcomes. In practice, however, it would be clearly naïve to assume that teachers would naturally select such well-matched learning objectives, ATs and TLAs given the learning outcome(s). A metric of how constructively aligned (or balanced) their selections are would therefore aid them in making alternative, better-suited, selections.

Before defining and computing such a metric using vectors and matrices, a further set of relations needs to be defined to represent the hierarchical relationships that exist between the four major components. Since the learning outcomes (W) are directly related to the learning objectives (X) which are subsequently directly related to both the TLAs (Z) and ATs (Y), the alignment between the learning outcomes and the TLAs and also between the learning outcomes and ATs are implicated. More specifically, the direct relationships are defined as follows:

- for each pair $w' \in W'$ and $x' \in X'$, $(w', x') \in V_1$ i.e. $w'V_1x'$
- for each pair $x' \in X'$ and $z' \in Z'$, $(x', z') \in V_2$ i.e. $x'V_2z'$
- for each pair $y' \in Y'$ and $x' \in X'$, $(y', x') \in V_3$ i.e. $y'V_3x'$

The following indirect (transitive) relationships are defined :

- $V_1$ and $V_2$ have x' in common which gives rise to the *composition of* $V_1$ and $V_2$ written as $V_1 \circ V_2$ and is defined by:

  $w'(V_1 \circ V_2)z'$ if for some $x' \in X'$ we have $w'V_1x'$ and $x'V_2z'$
- $V_1$ and $V_3$ have x' in common which gives rise to the *composition of* $V_1$ and $V_3$ written as $V_1 \circ V_3$ and is defined by:

  $w'(V_1 \circ V_3)y'$ if for some $x' \in X'$ we have $w'V_1x'$ and $x'V_3y'$
- $V_2$ and $V_3$ have x' in common which gives rise to the *composition of* $V_2$ and $V_3$ written as $V_2 \circ V_3$ and is defined by:

  $z'(V_2 \circ V_3)y'$ if for some $x' \in X'$ we have $x'V_2z'$ and $y'V_3x'$

The vectors and matrices required to compute an alignment metric can now be defined given the above relations. We need only compute an alignment metric for the *direct* relations i.e. individual metrics are computed for $V_1$, $V_2$ and $V_3$. The alignment of the transitive relations is by implication i.e. dependent on the alignment values $V_1$, $V_2$ and $V_3$.

To determine whether or not a module is constructively aligned we must compute the *degree* to which each of the three direct relations ($V_1$,$V_2$ and $V_3$) have reached their equilibrium. Note that $V_1$, $V_2$ and $V_3$ relations directly corresponds to $L^o$ trees, $L^b$ trees and AT trees respectively.

Assuming that the four major component sets and relations are available for a module, *full* module alignment is calculated as follows:

1. **Calculate the equilibrium value for relation $V_1$.**

This is achieved via the 7 following steps:

a. Assume the number of learning outcomes is fixed at *m* and as discussed earlier, we use the constant term $c_1$ to fix the valence (and thus vector-width) of $L^o$ trees. There must therefore be $c_1$ learning objectives per learning outcome. If $c_1$ objectives are not available for a given outcome then filler elements (i.e. <*empty_nodes*>) must be added to make up the number of dominated elements to $c_1$. The filler values are set to the level in Bloom's taxonomy indexed by the associated learning outcome (i.e. a value between 1 and 6 inclusive) to help maintain equilibrium.

b. Let **w** represent the set W' as a row vector such that each element of the vector represents a level in Bloom's taxonomy referenced by a corresponding learning outcome (stored in W).

$$\mathbf{w} = \begin{bmatrix} w_1' & w_2' & \dots & w_m' \end{bmatrix}^T$$

To recover the actual learning outcome definition we merely retrieve the left-hand side of the corresponding element in the relation R defined in i) above.

c. Let $D_1$ represent a matrix consisting of $c_1$ rows and $m$ columns, where each column corresponds to the set of suitable or 'desired' $c_1$ learning objectives (including filler elements) for a specific learning outcome. The crude assumption made for desired elements is that given a learning outcome $i$ the set of associated $c_2$ learning objectives should elicit the same cognitive ability from the student as the learning outcome. Since this assumption is made for all learning outcomes, the resulting target value is the summation of all such products. The author accepts that a semi-linear relationship would be more realistic whereby the learning objectives increase in complexity up to the level of the associated outcome and this is discussed later. Each value in $D_1$ refers to a level in Bloom's taxonomy (i.e., 1 to 6 inclusive) equal to that of the corresponding learning outcome. Matrix $D_1$ is defined as:

$$D_1 = \begin{bmatrix} d1_{11} & d1_{12} & \ldots & d1_{1i} & \ldots & d1_{1m} \\ d1_{21} & d1_{22} & \ldots & d1_{2i} & \ldots & d1_{2m} \\ & & & & & \\ d1_{j1} & d1_{j2} & \ldots & d1_{ji} & \ldots & d1_{jm} \\ & & & & & \\ d1_{c_1 1} & d1_{c_1 2} & \ldots & d1_{c_1 i} & \ldots & d1_{c_1 m} \end{bmatrix}$$

Let $\mathbf{d1}_i$ represent a column vector from matrix $D_1$ such that we refer to the set of $c_1$ 'desired' learning objectives associated with learning outcome $i$ as transposed and defined below:

$$\mathbf{d1}_i = \begin{bmatrix} d1_{i1} & d1_{i2} & \ldots & d1_{ic_1} \end{bmatrix}^T$$

d. Let $X_1$ represent a matrix consisting of $c_1$ rows and $m$ columns, where each column corresponds to the set of $c_1$ 'actual' learning objectives defined (explicitly or implicitly) by the teacher for a specific learning outcome (including filler elements). Since each non-filler value in $X_1$ refers to some $x'$ in $X'$ it is the actual level in Bloom's taxonomy referenced by the associated learning objective that is stored in the matrix.

$$X_1 = \begin{bmatrix} x'_{11} & x'_{12} & \ldots & x'_{1i} & \ldots & x'_{1m} \\ x'_{21} & x'_{22} & \ldots & x'_{2i} & \ldots & x'_{2m} \\ & & & & & \\ x'_{j1} & x'_{j2} & \ldots & x'_{ji} & \ldots & x'_{jm} \\ & & & & & \\ x'_{c_1 1} & x'_{c_1 2} & \ldots & x'_{c_1 i} & \ldots & x'_{c_1 m} \end{bmatrix}$$

Also, let $\mathbf{x1}_i$ represent a column vector from matrix $X_1$ such that we refer to the set of $c_1$ teacher-defined learning objectives associated with learning outcome $i$ as defined below:

$$x1_i = \begin{bmatrix} x'_{i1} & x'_{i2} & \ldots & x'_{ic_1} \end{bmatrix}^T$$

e. Calculate the alignment values between the learning outcomes and learning objectives as follows:

- Calculate the *desired alignment* value using the *inner dot product* between each learning outcome, $w_i$, and its corresponding set of $c_1$ desired learning objectives stored in matrix $D_1$:

$$t1_i = w'_i d1_{1i} + w'_i d1_{2i} + \ldots w'_i d1_{ji} \ldots + w'_i d1_{c_1 i} = \sum_{j=1}^{c_1} w'_i d1_{ji}$$

where $i$ = 1..m.

Since we are making the naïve assumption that each desired objective elicits the same level as the associated outcome then $t1_i = c_1 * w_i'^2$

- Compute the *actual alignment* value, $\mathbf{u1}_i$, using the inner dot product between each learning outcome, $w'_i$, and its corresponding set of $c_1$ actual learning objectives stored in matrix $X_1$:

$$u1_i = \sum_{j=1}^{c_1} w'_i x1'_{ji} \quad \text{where } i = 1..m.$$

f. Calculate the difference or *misalignment* between the desired and actual alignment values for each individual learning outcome. Let **e1** refer to the vector of misalignment between the learning outcomes and learning objectives, where the misalignment value for learning outcome *i* is defined as:

$e1_i = u1_i - t1_i$ where *i* = 1..m.

The absolute values of **e1**, denoted $\left|\mathbf{e1}_i\right|$, are used to compute alignment and ignores the arithmetic sign of alignment values in favour of magnitude. This allows us to measure according to the magnitude of alignment and is illustrated further using the following piece of structured English:

```
For each learning outcome i (i=1..m)
 Do
  If $|\mathbf{e1}_i| <= \tau$
   Then If one or more $x'_{ji} = w'_i$ (for each j)
          Then the learning objectives are aligned with learning outcome i
  Else If $|\mathbf{e1}_i| > \tau$ AND $\mathbf{e1}_i > 0$
        Then If one or more $x'_{ji} = w'_i$ (for each j)
               Then the learning objectives are positively misaligned with learning outcome i
  Else
        The learning objectives are negatively misaligned with learning outcome i
```

Where $\tau$ is a threshold value defined *a priori* and determines the level of acceptable error. The author uses the term '*positively misaligned*' to refer to the situation where a teacher has prescribed learning objectives that elicit cognitive abilities from the student that collectively exceeds that required by the associated module learning outcome. It is considered positive in that the student will still be able to meet the learning outcomes if the learning objectives are achieved. For either actual alignment or *positive misalignment* to occur, at least one learning objective must elicit the same level of cognitive ability from the student as required by the associated learning outcome.

Conversely, the term '*negatively misaligned*' refers to a state where the teacher has defined learning objectives that elicit lower cognitive abilities from the student than that defined in the learning outcome. Clearly, it is referred to as negative as even if the students achieve all of the learning objectives the learning outcome itself is still unobtainable.

g. Finally, calculate the $V_1$ equilibrium value to measure the *overall* alignment between the learning outcomes and the actual learning objectives assigned to them. This is obtained by calculating the root mean squared error (RMSE) across all elements of **e1**. The alignment errors for each learning outcome are squared to maintain the magnitude of each misalignment value irrespective of sign and obviously to avoid negative error values from cancelling out positive error values. The average error is then computed. Finally, the squared root of the resulting value provides an alignment value representative of the different misalignment errors stored in **e1**. This calculation is expressed as follows:

$$V_1\_equilibrium = \sqrt{\frac{1}{m}\sum_{i=1}^{m} e1_i^{\,2}}$$

## 2. Calculate the equilibrium value for relation $V_2$.

The same 7-step process described for $V_1$ is used and can be summarised for $V_2$ as follows:

a. As discussed earlier, we use the constant term $c_2$ to fix the valence of $L^b$ trees. There must therefore be $c_2$ TLAs for each of the *n* learning objectives defined (including filler elements).

b. Let **x2** represent the vector of all *n* teacher-defined learning objectives (no filler elements), where all elements within the vector represent a level in Bloom's taxonomy and thus a value between 1 and 6. To recover the actual learning objective definition, we retrieve the left-hand side of the corresponding element in the relation S.

c. Let $D_2$ represent a matrix consisting of $c_2$ rows and *n* columns, where each column corresponds to the set of 'desired' $c_2$ TLAs for each learning objective. Let $\mathbf{d2}_j$ represent a column vector from matrix $D_2$ such that we refer to the set of $c_2$ 'desired' *TLAs* associated with learning objective *j*.

d. Let $Z_1$ represent a matrix consisting of $c_2$ rows and *n* columns, where each row corresponds to the actual set of $c_2$ TLAs (including filler elements) used to help students achieve a specific learning objective.

e. Calculate the $V_2$ alignment values as follows:

- Calculate the desired alignment value, $\mathbf{t2}_j$, using the inner product between each learning objective, $x2_j$, and its corresponding set of *q* desired TLAs stored in matrix $D_2$. Since we are making the naïve assumption that each desired TLA elicits the same level of ability as the associated objective then $t2_j = c_2 * x2_j^2$

- Compute the actual alignment value, **u2**$_j$, using the inner dot product between each learning objective, $x2_j^{'}$, and its corresponding set of $c_2$ TLAs stored in matrix $Z_1$.

f. Calculate the difference or *misalignment* between the desired and actual alignment values for each individual learning objective i.e.

$e2_j = u2_j - t2_j$ where $j$ = 1..n.

As before, the absolute values of vector **e2** values indicate the degree to which each *element* of $V_2$ is aligned as shown in the following logic below:

```
For each learning objective j (j=1..n)
  Do
    If $|\mathbf{e2}_j| <= \tau$
      Then If one or more $z'_{kj} = x2_j$ (for each k)
        Then the TLAs are aligned with learning objective j
    Else If $|\mathbf{e2}_j| > \tau$ and $\mathbf{e2}_j > 0$
      Then If one or more $z'_{kj} = x2_j$ (for each k)
        Then the TLAs are positively misaligned with learning objective j
    Else
      The TLAs are negatively misaligned with learning objective j
```

g. Calculate the $V_2$ equilibrium value as follows to measure the *overall* alignment between the learning objectives and TLAs:

$$V_2\_equilibrium = \sqrt{\frac{1}{n}\sum_{j=1}^{n} e2_j^{\,2}}$$

### 3. Calculate the equilibrium value for relation $V_3$.

The same 7-step process as above is used and can be summarised for $V_3$ as follows:

a. We use the constant term $c_3$ to fix the valence of AT trees. There must therefore be $c_3$ learning objectives for each of the $p$ ATs used (including filler values where necessary).

b. Let **y** represent the set $Y'$ as a row vector such that each element of the vector represents a level in Bloom's taxonomy referenced by a corresponding AT (stored in Y).

c. Let $D_3$ represent a matrix consisting of $c_3$ rows and $p$ columns, where each column corresponds to the set of suitable or 'desired' $c_3$ learning objectives (including filler elements) assessed by a specific AT. Each value in $D_3$ refers to a level in Bloom's taxonomy equal to that of the corresponding AT. Let **d3**$_l$ represent a column vector from matrix $D_3$ such that we refer to the set of $c_3$ 'desired' learning objectives associated with learning outcome *l*.

d. Let $X_2$ represent a matrix[3] consisting of $c_3$ rows and $p$ columns, where each column corresponds to the set of $c_3$ 'actual' learning objectives (including filler elements) assessed by the teacher using a specific AT. Also, let **x3**$_l$ represent a column vector from matrix $X_2$ such that we refer to the set of $c_3$ teacher-defined learning objectives assessed by AT *l*.

e. Calculate the alignment values between an AT and its associated learning objectives as follows:

- Calculate the desired alignment value using the inner dot product between each AT, $y_l^{'}$, and its corresponding set of $c_3$ desired learning objectives stored in matrix $D_3$. Since we are making the naïve assumption that each desired objective elicits the same level as the associated outcome then $t3_l = c_3 * y_l^{'2}$
- Compute the *actual alignment* value, **u3**$_l$, using the inner dot product between each AT, $y_l^{'}$, and its corresponding set of $c_3$ actual learning objectives stored in matrix $X_2$.

f. Calculate the difference or *misalignment* between the desired and actual alignment values for each individual AT i.e. $e3_l = u3_l - t3_l$ where $l$ = 1..p.

As before, the absolute values of vector **e3** elements indicate the degree to which each *element* of $V_3$ is aligned as shown in the following logic below:

```
For each AT_l (l =1..p)
  Do
    If $|\mathbf{e3}_l| <= \tau$
      Then If one or more $x'_{jl} = y'_l$ (for each j)
        Then the learning objectives are aligned with AT_l
```

---

[3] Note that this refers to a matrix and not a vector and is thus very different to x2 used to reference a previous vector.

```
Else If $|e3_l| > \tau$ AND $e3_l > 0$
    Then If $x'_{jl} = y'_l$ (for each $j$)
        Then the learning objectives are positively misaligned with $AT_l$
  Else
    The learning objectives are negatively misaligned with $AT_l$
```

g. Calculate the $V_3$ equilibrium value as follows to measure the *overall* alignment between the learning objectives and ATs:

$$V_3\_equilibrium = \sqrt{\frac{1}{p}\sum_{l=1}^{p} e3_l^{\,2}}$$

4. **Calculate the overall equilibrium value.**

The equilibrium value consolidating all direct relations and representing constructive alignment for the whole module design is simply:

$$\frac{V_1\_equilibrium + V_2\_equilibrium + V_3\_equilibrium}{3}$$

If each element of $e1_i$, $e2_j$ and $e3_l$ has been classified as either ‘aligned’ or ‘positively misaligned’ then it could be broadly stated that the module as a whole is fully *constructively aligned* otherwise there is some misalignment between the four components of the teaching system. Clearly, in order to determine the cause of any misalignment then the result of the inner dot products for the individual relations ($V_1$,$V_2$ and $V_3$) needs to be examined in order to trace mismatching elements. A worked example is provided in Appendix B.

## 3. DISCUSSION

The model presented meets its stated aims in that it:

- provides a quantitative measure of alignment between individual system components and of full constructive alignment for an entire module;
- potentially facilitates teaching practitioners to adapt their practice to better align their modules by making them aware of alignments and misalignments within their educational designs.

Also, it is possible to extend the applicability of the model by enriching information stored within nodes of $L^o$, $L^b$, and AT trees to help practitioners develop fair educational designs that enforce inclusivity (e.g. to support students with disabilities). Since alignment is verb-based we can exploit the powerful features of syntactic theory to generate and enforce well-defined alignment structures. For example, a common phenomenon reported in linguistics is that of *verb subcategorisation* [21] whereby different types of verbs require or ‘subcategorise for’ different patterns of arguments such as prepositional phrases and object noun phrases. A transitive verb, such as ‘slap’, requires an object noun phrase to refer to the agent, which is acted upon by the subject e.g. *Jill slapped Jack*. Further, transitive verbs such as ‘put’ require an additional prepositional phrase to indicate position of the object noun e.g. *Jill put the bucket down*. Intransitive verbs such as ‘sleep’ and ‘run’, on the other hand, do not require object noun phrases e.g., *Jill slept.* The computational model proposed can utilise such principles to enforce selectional restrictions based on learning elicited and environmental constraints. For example, a particular learning objective utilising a verb such as ‘apply’ will restrict the type of TLAs and ATs that can be used to one or more that elicits or assesses (individually or collectively) the cognitive skill of *application*. Environmental restrictions, such as available rooms, resource or student disability, may further reduce the types of allowable TLAs and ATs.

The model has only been evaluated on a number of semester-long modules and could be considered idealistic in its current form. For the model to scale-up to more realistic contexts, we identify four areas requiring further research, these are: a) adequacy of Bloom’s taxonomy for categorising system components; b) establishing ‘desired’ objectives, TLAs and ATs; c) acceptable values for the alignment threshold, $\tau$ and finally d) usefulness of alignment metric as tree complexity increases.

Also, for teachers to be able to use the model in its current form, it is assumed that teachers’ know a priori what the main components of an educational design are and how they relate. It is envisaged, however, that a complete software implementation of the model will aid the practitioner in identifying and relating components (symbolically via a graphical user interface) whilst abstracting them away from the actual alignment computations used to determine the alignment measures.

## 4. CONCLUSIONS

Biggs [3] states that any discussions about good teaching should include that of alignment models. Biggs integrates instructional design with constructivist principles to produce a framework, referred to as constructive alignment that systematically operationalises the important characteristics of a good teaching practitioner, which are to:

- be able to define what the teacher wants the student to learn and achieve (learning outcomes);
- be able to define what students have to do to demonstrate they have learned the objectives to the required level (assessment tasks);
- be aware of the different cognitive skills each of the teaching and learning activities elicit from the student and be able to instantiate them according to the learning objectives defined (student-centred teaching and learning activities).

The computational mode presented in this paper utilises vectorial representations and computations to provide numerical measures of alignment for both holistic and individual aspects of an educational design. A structural and generative perspective of alignment systems is adopted to enable relationships across the above desiderata for good teaching to be represented and manipulated in vectorial form. Crucially, the computation of the alignment metrics is dependent upon three important factors: i) the ability to accurately cluster outcomes, objectives, ATs and TLAs according to the level of cognitive ability they elicit or assess; ii) a priori definitions of acceptable prototypes of perfect or 'desired' alignment values from which to 'benchmark' against and iii) defining realistic alignment threshold values. Although, further research is required on all three counts, the model is a significant step towards the realisation of a software tool to facilitate teachers to systematically and consistently produce and manage constructively aligned programmes of teaching and learning.

It is envisaged that the model will have broad appeal as it allows the quality of educational designs to be measured and works on the principle of 'practice techniques' and 'learning elicited' as opposed to content. In fact, we hope this metric becomes one of many that measures any phenomenon associated with educational learning, perhaps part of a wider field called "*educametrics*" ?

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# APPENDIX A: ALIGNMENT TABLES

The level in Bloom's taxonomy assigned for each AT in figure A1 and TLA in figure A2 is based on the information provided in Biggs [4]. It is not a precise grouping and as Biggs noted, research into such groupings is so far incomplete and much work still needs to be done.

| **Assessment type and task** | **Type of learning assessed** | **Bloom's tax. (1-6)** |
|---|---|---|
| *1. Extended prose, essay-type ($AT^e$)* | | |
| essay exam ($AT^e_1$) | rote, question spotting, speed structuring | 5 |
| open book ($At^e_2$) | as above but less memory and greater coverage | 2 |
| assignment, take-home ($AT^e_3$) | read widely, interrelate, organise, apply, copy | 5 |
| *2. Objective test ($AT^o$)* | | |
| multiple-choice ($AT^o_1$) | recognition, strategy, comprehension, coverage | 2 |
| ordered outcome ($AT^o_2$) | hierarchies of understanding | 3 |
| *3. Performance assessment ($AT^p$)* | | |
| practicum ($AT^p_1$) | skills needed in real life, procedural knowledge | 4 |
| seminar, presentation ($AT^p_2$) | communication skills | 3 |
| posters ($AT^p_3$) | Concentrating on relevance, application | 3 |
| interviewing ($AT^p_4$) | responding interactively, recall, application | 3 |
| critical incidents ($AT^p_5$) | reflection, application, sense of relevance | 6 |
| project ($AT^p_6$) | application, research, problem solving | 4 |
| reflective journal ($AT^p_7$) | reflection, application, sense of relevance | 6 |
| case study, problems ($AT^p_8$) | application, professional skills | 3 |
| portfolio ($AT^p_9$) | reflection, creativity, unintended outcomes | 6 |
| *4. Rapid ATs (large class) ($AT^r$)* | | |
| concept maps ($AT^r_1$) | coverage, relationships, some holistic understanding | 5 |
| Venn diagrams ($AT^r_2$) | Relationships | 2 |
| three-minute essay ($AT^r_3$) | level of understanding, sense of relevance | 3 |
| gobbets ($AT^r_4$) | realising importance of significant detail, some multistructural understanding across topics | 2 |
| short answer ($AT^r_5$) | recall units of information, coverage | 2 |
| letter to a friend ($AT^r_6$) | holistic understanding, application, reflection | 3 |
| cloze ($AT^r_7$) | Comprehension of main ideas | 2 |

Figure A1. Assessment tasks and the types of learning assessed by those tasks (adapted from Biggs, 1999). Assessment task coding scheme ($AT^{type}_n$) and index to highest level in Bloom's taxonomy added.

| **T LA** | **A form of learning** | **Bloom's tax. (1-6)** |
|---|---|---|
| *1. Teacher-controlled (TLA$^{t}$)* | | |
| lecture, set texts (TLA$^{t}_{1}$) | reception of selected content | 2 |
| think aloud (TLA$^{t}_{2}$) | demonstrate conceptual skills | 3 |
| questioning (TLA$^{t}_{3}$) | clarifying, seeking error | 4 |
| advance organizer (TLA$^{t}_{4}$) | structuring, preview | 5 |
| concept mapping (TLA$^{t}_{5}$) | structuring, overview | 5 |
| tutorial (TLA$^{t}_{6}$) | elaboration, clarification | 2 |
| laboratory (TLA$^{t}_{7}$) | procedures, application | 4 |
| excursion (TLA$^{t}_{8}$) | experiential knowledge, interest | 2 |
| seminar (TLA$^{t}_{9}$) | clarify, presentation skill | 3 |
| *2. Peer-controlled (TLA$^{p}$)* | | |
| various groups (TLA$^{p}_{1}$) | elaboration, problem-solving, metacognition | 6 |
| learning partners (TLA$^{p}_{2}$) | resolve differences, application | 6 |
| peer teaching (TLA$^{p}_{3}$) | depends whether teacher or taught | 3+? |
| spontaneous collaboration (TLA$^{p}_{4}$) | breadth, self-insight | 3+? |
| *3. Self-controlled (TLA$^{s}$)* | | |
| generic study skills (TLA$^{s}_{1}$) | basic self-management | 5/6 |
| content study skills (TLA$^{s}_{2}$) | information handling | 5/6 |
| metacognitive learning skills (TLA$^{s}_{3}$) | independence and self-monitoring | 6 |

Figure A2. Teaching and learning activities and the types of learning they elicit. Adapted from Biggs [4]. TLA coding scheme (TLA$^{type}_{n}$) and index to highest level in Bloom's taxonomy added.

# APPENDIX B: A WORKED EXAMPLE

The worked example is for one of the author's undergraduate computing modules, Introduction to Information Systems (IIS). IIS accounts for 10 credit points of a degree programme and is run in the first Semester. It is a compulsory level 1 module of all Computing degree programmes administered by the author's School. The assessed learning outcomes for IIS are shown in figure B1. Each outcome is linked to an associated level in Bloom's taxonomy based on the main active verb and is shown in parentheses.

| **Introduction to Information Systems Module : Assessed Learning Outcomes** | | |
|---|---|---|
| $L^{o}_{1}$ | Explain the role and skills of the Systems Analyst. | (2) |
| $L^{o}_{2}$ | Explain Systems Development Lifecycles and Methodologies. | (2) |
| $L^{o}_{3}$ | Formulate a set of balanced data flow diagrams (DFDs) for a simple information system. | (5) |
| $L^{o}_{4}$ | Specify the processing logic for simple DFD processes using a logic modelling technique. | (5) |
| $L^{o}_{5}$ | Formulate a normalised data model (to third normal form) showing entities, entity attributes, entity relationships and data dictionary entries for a simple information system. | (5) |

Figure B1. Module learning outcomes.

The IIS module framework is summarised as follows:

**Class:** 250 first-year undergraduate computing students.

**Teaching structure (per week):** one plenary lecture, one tutorial of 10 groups of 25 students - all classes are thus evidently large. There are eight major topics introduced and variously elaborated in the lectures and tutorials over the 12-week semester. A reading schedule is given and students are expected to produce questions to be answered during lectures and tutorials. Each lecture and corresponding tutorial explicitly has a set of teacher-defined learning objectives which the students must achieve to meet the associated learning outcome.

**Staff:** One senior lecturer, who is the module leader and responsible for: creating all teaching and learning materials, delivering lectures, taking some tutorials, managing assessment marking, moderation and reporting of results to administration; three teaching assistants who between them take the remaining tutorials and help with assessment.

**Summative Assessment:** coursework worth 30% of the module and consists of an individual MC test and 3 group take-home assignments to be worked on between tutorials (students are informed a priori to encourage preparation); 2 hour individual examination worth 70% of the module which addresses the learning objectives associated with the group coursework.

For reasons of brevity, the worked example will compute alignment for a single learning outcome, *learning outcome 3* or $L^{o}_{3}$. This will be sufficient to show how the computational model works with a real module specification. It is envisaged that the reader will then find it intuitive to extend the example to an entire module given the repetitious nature of the computations.

Let us define the appropriate sets for $L^{o}_{3}$ as follows:

W = {"$L^{o}_{3}$ – *Formulate a set of balanced Data Flow Diagrams (DFDs) for a simple information system*"}

$n(W) = m = 1$

$W^{'} = \{5\}$

Let us define the associated set of learning objectives as:

X = {"$L^b_1$ – *Define systems modelling and differentiate between logical and physical system models*",
"$L^b_2$ – *Define process models and describe its benefits*",
"$L^b_3$ – *Demonstrate an understanding of the basic concepts and constructs of a DFD*",
"$L^b_4$ – *Explain the differences among four types of DFDs: current physical, current logical, new physical and new logical*",
"$L^b_5$ – *Formulating level-0 (context) and level 1 DFDs*",
"$L^b_6$ – *Decompose DFDs into lower-level diagrams (DFD levelling)*",
"$L^b_7$ – *Demonstrate an understanding of DFD balancing*"
}

*n*(X) = n = 7

$X'$= {1, 1, 3, 2, 5, 4, 3}

Since we are computing alignment for one outcome, $L^o_3$, then $L^o_3$ simply "dominates" all of the objectives in X' (i.e. $w'V_1x'$) and represents the $L^o_3$ tree.

Assume that Z is the set of all TLAs used to encourage students to achieve the objectives in X:

Z = {$TLA^t_1$, $TLA^t_2$, $TLA^t_3$, $TLA^t_4$, $TLA^t_6$, $TLA^t_9$, $TLA^p_1$, $TLA^p_2$}

*n*(Z) = q = 8

Z' = {2, 3, 4, 5, 2, 3, 4, 3}

(Note that the TLA coding scheme from table A.2 is used for clarity).

Since an objective may be associated with multiple TLAs, the direct relationship between X' and Z' (i.e. $x'V_2z'$) requires us to define $V_2$ as being a *class* of sets whereby each element of $V_2$ refers to a subclass of Z'. Each element in X represents a parent node and the corresponding element in $V_2$ represents the children nodes therefore forming $L^b$ trees. For clarity, the actual TLA code is used to express association (z) rather than the level of learning elicited (z') as different TLAs can elicit the same level of learning. Each element of $V_2$ is therefore:

$$V_2 = \begin{bmatrix} \{TLA_1^t, TLA_2^t, TLA_3^t, TLA_6^t\}, \\ \{TLA_1^t, TLA_2^t, TLA_3^t, TLA_6^t\}, \\ \{TLA_6^t, TLA_9^t\}, \\ \{TLA_1^t, TLA_6^t, TLA_2^p\}, \\ \{TLA_1^t, TLA_2^t, TLA_3^t, TLA_4^t, TLA_1^p, TLA_2^p\}, \\ \{TLA_1^t, TLA_2^t, TLA_3^t, TLA_4^t, TLA_1^p, TLA_2^p\}, \\ \{TLA_6^t, TLA_9^t\} \end{bmatrix}$$

Assume that Y is the ordered set of all ATs used to assess students' ability to achieve the learning objectives stated in X:

$Y = \{AT^r{}_5, AT^p{}_2, AT^e{}_1, AT^e{}_3\}$

$n(Y) = p = 4$

$Y' = \{2, 3, 5, 5\}$

(Note that the AT coding scheme from table A.1 is used for clarity).

Multiple objectives may be associated with each assessment task, the direct relationship between Y' and X' (i.e. $V_3$) therefore requires us to define $V_3$ as being a class of ordered sets whereby each element of $V_3$ refers to a subclass of X'. $V_3$ is an ordered set in that the first element in $V_3$ corresponds to the first element in Y and so on. Moreover, each element in Y represents a parent node and the corresponding element in $V_3$ represents the children nodes therefore forming AT trees. For clarity, the actual learning objective code ($L^b{}_j$) is used to express association (x) rather than the level of learning elicited (x') as different active verbs can elicit the same level of learning. Each element of $V_3$ is therefore:

$$V_3 = \begin{bmatrix} \{L_1^b, L_2^b, L_4^b\}, \\ \{L_3^b, L_7^b\}, \\ \{L_5^b, L_6^b\}, \\ \{L_5^b, L_6^b\} \end{bmatrix}$$

Now that the four major component sets and relations have now been defined we need only set the alignment threshold, $\tau$. For this example, assume that $\tau$ is set to 30. Alignment for $L^o{}_3$ can now be calculated in four main steps as follows:

1. **Calculate the equilibrium value for relation $V_1$.**
   a. Recall that we use the constant term $c_1$ to fix the valence of $L^o$ trees. We set $c_1$ to 7 as we are only computing alignment for one $L^o$ that dominates seven $L^b$s.

   b. Let **w** represent the set $W'$ as a row vector such that:

   $$\mathbf{w} = [5]$$

   In this case, **w** is actually a scalar.

   c. Let **d1** represent the vector of $c_1$ 'desired' learning objectives for $L^o{}_3$ and **x1** represent the vector of $c_1$ 'actual' learning objectives:

$$\mathbf{d1} = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \end{bmatrix} \quad x\mathbf{1} = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \\ 5 \\ 4 \\ 3 \end{bmatrix}$$

d. Calculate the alignment values between the learning outcomes and learning objectives as follows:

- Calculate the *desired alignment* value:

$$t1 = \sum_{j=1}^{c_1} w' d1_j = 175$$

- Compute the *actual alignment* value:

$$u1 = \sum_{j=1}^{c_1} w' x1_j = 95$$

e. Calculate the difference or *misalignment* between the desired and actual alignment value for $L^o_3$ represented by **e1**:

$$e1 = u1 - t1 = -80$$

Since $|\mathbf{e1}|$ is greater than $\tau$ and e1 is negative $V_1$ is *negatively misaligned*.

f. Calculate the $V_1$_equilibrium value:

$$V_1\_equilibrium = \sqrt{\frac{1}{m}\sum_{i=1}^{m} e1_i^{\,2}} = 80$$

The $V_1$_equilibrium is considerably higher than $\tau$ and would typically indicate a significant imbalance between the outcome and its objectives. After assessing the outcome/objective associations, however, this level of disequilibrium can be apportioned to the $D_1$ matrix in that unrealistic ‘desired’ values are given. As mentioned previously, a semi-linear relationship between components would be more realistic.

**2. Calculate the equilibrium value for relation $V_2$.**

a. Recall that we use the constant term $c_2$ to fix the valence of $L^b$ trees. We set $c_2$ to 6, as it is the maximum number of TLAs dominated by a single $L^b$. Filler elements are equal to the parent $L^b_j$ value.

b. Let **x2** represent the vector of all *n* learning objectives from X’:

$$\mathbf{x2} = [1 \quad 1 \quad 3 \quad 2 \quad 5 \quad 4 \quad 3]^T$$

To retrieve actual objective we obtain the corresponding element from X.

c. Let $D_2$ represent the matrix consisting of $c_2$ rows and $n$ columns, where each column corresponds to the set of 'desired' $c_2$ TLAs for each learning objective. Matrix $D_2$ is defined as:

$$D_2 = \begin{bmatrix} 1 & 1 & 3 & 2 & 5 & 4 & 3 \\ 1 & 1 & 3 & 2 & 5 & 4 & 3 \\ 1 & 1 & 3 & 2 & 5 & 4 & 3 \\ 1 & 1 & 3 & 2 & 5 & 4 & 3 \\ 1 & 1 & 3 & 2 & 5 & 4 & 3 \\ 1 & 1 & 3 & 2 & 5 & 4 & 3 \end{bmatrix}$$

Let $\mathbf{d2}_j$ represent a column vector from matrix $D_2$ such that we refer to the set of $c_2$ 'desired' *TLAs* associated with learning objective *j* as transposed and defined below:

$$\mathbf{d}2_j = [d2_{j1} \quad d2_{j2} \quad \dots \quad d2_{jc_2}]^T$$

d. Let $Z_1$ represent a matrix consisting of $c_2$ rows and $n$ columns, where each row corresponds to the actual set of $c_2$ TLAs (including filler elements) used to help students achieve a specific learning objective. $Z_1$ is defined as:

$$Z_1 = \begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 2 & 3 & 3 & 3 \\ 4 & 4 & 3 & 3 & 4 & 4 & 3 \\ 2 & 2 & 3 & 2 & 5 & 5 & 3 \\ 1 & 1 & 3 & 2 & 4 & 4 & 3 \\ 1 & 1 & 3 & 2 & 3 & 3 & 3 \end{bmatrix}$$

e. Calculate the $V_2$ alignment values as follows:

- Calculate the *desired alignment* values:

$$\mathbf{t2}_j = \sum_{k=1}^{c_2} x2_j d2_{kj}$$

$$\mathbf{t2} = [6 \quad 6 \quad 54 \quad 24 \quad 150 \quad 96 \quad 54]$$

- Compute the *actual alignment* values:

$$u2_j = \sum_{k=1}^{c_2} x2_j z'_{kj}$$

$$\mathbf{u2} = \begin{bmatrix} 13 & 13 & 51 & 26 & 105 & 84 & 51 \end{bmatrix}$$

f. Calculate the misalignment between the desired and actual alignment values:

$$e2_j = u2_j - t2_j$$

$$e\mathbf{2} = \begin{bmatrix} 7 & 7 & -3 & 2 & -45 & -12 & -3 \end{bmatrix}$$

Since $|\mathbf{e}2_5|$ is greater than $\tau$ and $\mathbf{e2}_5$ is negative, the "$L^b_5$ tree" is negatively misaligned. All other $L^b$ trees are aligned since the absolute error values are less than $\tau$ and at least one TLA from each of the other trees elicit the required cognitive skill.

g. Calculate the $V_1$_equilibrium value:

$$V_2\_equilibrium = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e2_j^{\;2}} = 18.1$$

$V_2$_equilibrium is less than $\tau$ and could generally be considered aligned. The high misalignment value, however, indicates that some learning objective/TLA associations need to be assessed and modified to further reduce this disequilibrium.

**3. Calculate the equilibrium value for relation $V_2$.**

a. Recall that we use the constant term $c_3$ to fix the valence of AT trees. We set $c_3$ to 3, as it is the maximum number of $L^b$s dominated by a single AT. Filler elements are equal to the parent AT value.

b. Let **y** represent the set $Y'$ as a row vector:

$$y = \begin{bmatrix} 2 & 3 & 5 & 5 \end{bmatrix}^T$$

c. Let $D_3$ represent a matrix consisting of $c_3$ rows and $p$ columns, where each column corresponds to the set of suitable or 'desired' $c_3$ learning objectives (including filler elements) assessed by a specific AT:

$$D_3 = \begin{bmatrix} 2 & 3 & 5 & 5 \\ 2 & 3 & 5 & 5 \\ 2 & 3 & 5 & 5 \end{bmatrix}$$

Let $\mathbf{d3}_l$ represent a column vector from matrix $D_3$ such that we refer to the set of $c_3$ 'desired' learning objectives associated with learning outcome $l$ as transposed and defined below:

$$\mathbf{d}3_l = \begin{bmatrix} d3_{l1} & d3_{l2} & \dots & d3_{lc_3} \end{bmatrix}^T$$

d. Let $X_2$ represent a matrix consisting of $c_3$ rows and $p$ columns, where each column corresponds to the set of $c_3$ 'actual' learning objectives (including filler elements) assessed by the teacher using a specific AT. $X_2$ is defined as follows:

$$X_2 = \begin{bmatrix} 1 & 3 & 5 & 5 \\ 1 & 3 & 4 & 4 \\ 2 & 3 & 5 & 5 \end{bmatrix}$$

Also, let $\mathbf{x3}_l$ represent a column vector from matrix $X_2$ such that we refer to the set of $c_3$ teacher-defined learning objectives assessed by AT $l$ as defined below:

$$x3_l = \begin{bmatrix} x'_{l1} & x'_{l2} & \dots & x'_{lc_3} \end{bmatrix}^T$$

e. Calculate the alignment values between an AT and its associated learning objectives as follows:

- Calculate the *desired* alignment:

$$t3_l = \sum_{j=1}^{c_3} y'_l d3_{jl}$$

$$\mathbf{t}3 = \begin{bmatrix} 12 & 27 & 75 & 75 \end{bmatrix}$$

- Compute the *actual* alignment value:

$$u3_l = \sum_{j=1}^{c_3} y'_l x'_{jl}$$

$$u3 = \begin{bmatrix} 8 & 27 & 70 & 70 \end{bmatrix}$$

f. Calculate the difference or *misalignment* between the desired and actual alignment values for each individual AT:

$$e3_l = u3_l - t3_l$$

$$e3 = \begin{bmatrix} -4 & 0 & -5 & -5 \end{bmatrix}$$

All AT trees are aligned since all absolute error values are less than $\tau$ and at least one $L^b$ within each AT tree elicits the required level of cognitive ability to be assessed.

g. Calculate the $V_3$ equilibrium value:

$$V_3\_equilibrium = \sqrt{\frac{1}{p}\sum_{l=1}^{p} e3_l^{\,2}} = 4.1$$

$V_3$_equilibrium is less than $\tau$ and is considered aligned. Clearly, the desired and actual objective vectors are much more similar than those for the $V_1$ and $V_2$ alignment computations. This similarity is reflected numerically in **e3** and $V_3$_equilibrium.

**4. Calculate the overall equilibrium value.**

The equilibrium value consolidating all direct relations and representing constructive alignment, in this case for a single outcome of a module, is simply:

$$\frac{V_1\_equilibrium + V_2\_equilibrium + V_3\_equilibrium}{3}$$

$$\frac{80 + 18.1 + 4.1}{3} = 34.1$$

If each element of $e1_i$, $e2_j$ and $e3_l$ has been classified as either being ‘aligned’ or ‘positively misaligned’ then it could be broadly stated that the module is *constructively aligned* for the learning outcome addressed. Clearly, $V_1$ is responsible for the majority of misalignment and thus disequilibrium and it is envisaged that the practitioner would first inspect the relationships between the learning outcome and the objectives they had subsequently prescribed. The significance of the magnitude for the individual error and equilibrium values requires more research for it to be considered an accurate and truly representative measure of alignment.